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SIMULATION CONSULTANTS

advanced

simulations

that are

fast
accurate
useful

IS THERE A UNIVERSAL FATIGUE MODEL?

Udayan Kanade
CEO, Noumenon

SAE FD&E 2017 Spring Meeting
April 25th 2017

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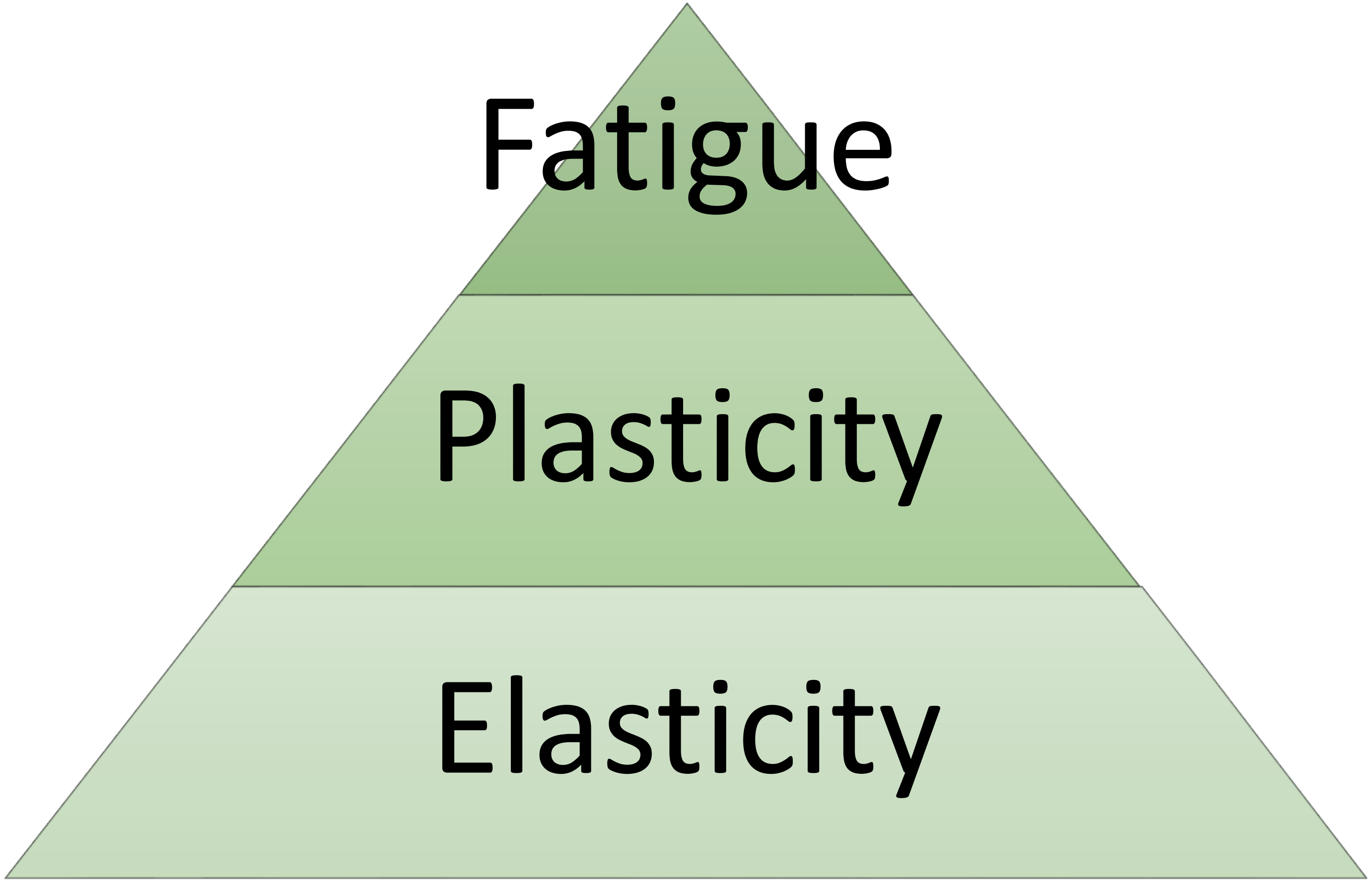
Today's presentation



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Universal Elasticity

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Many elasticity models

Linear elasticity
Mooney-Rivlin
Generalized Rivlin
Arruda-Boyce
Ogden
Saint Venant-Kirchhoff
Fung
Yeoh
Marlow
Neo-Hookean
Gent
…

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Assumptions
Approximations
Observations

Many elasticity models

Linear elasticity
Mooney-Rivlin
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Universal elasticity model

Basic Laws of Physics

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Universal elasticity model

Basic Laws of Physics

Conservation of mass

Newton's 2nd law ($\dot{P} = F$)

Elasticity

$$\rho_M \frac{\partial^2 \phi}{\partial t^2} = \nabla \cdot P(\nabla_X \phi) + \rho_M b$$

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$$\rho_M \frac{\partial^2 \phi}{\partial t^2} = \nabla \cdot P(\nabla_X \phi) + \rho_M b \qquad \mathbb{R}^9 \to \mathbb{R}^9$$

$$P(F) = \psi\left(FF^T\right)F \qquad \mathbb{R}^6 \to \mathbb{R}^6$$

$$\psi\left(U\Sigma U^T\right) = U\psi(\Sigma)U^T \qquad \mathbb{R}^3 \to \mathbb{R}^3$$

$$\psi(\Sigma) = 2\frac{dE}{d\Sigma} \qquad \mathbb{R}^3 \to \mathbb{R}$$

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Modeling the material

- Inverse Methods
- Uncertainty quantification

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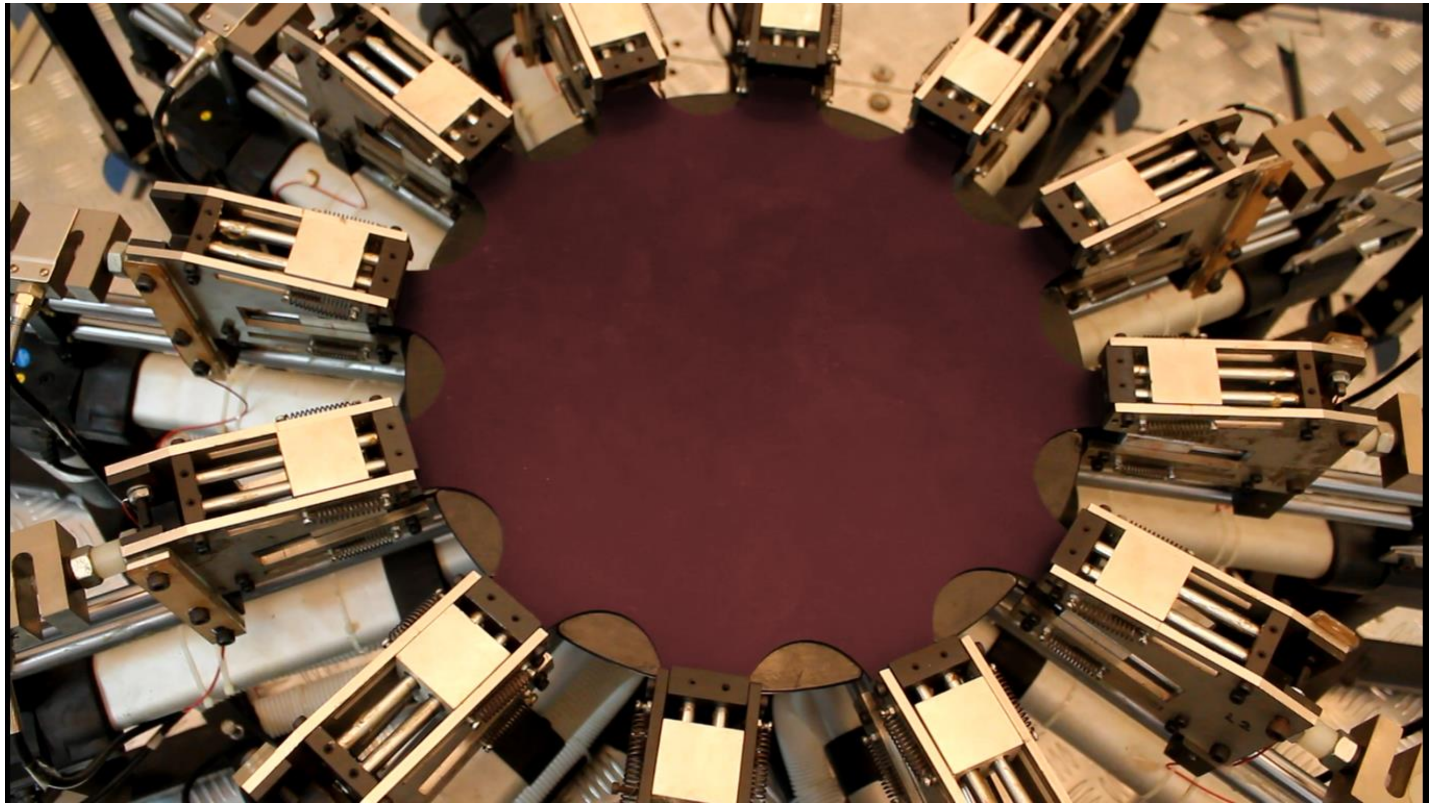
Mooney-Rivlin machine

- Inverse Methods
- Uncertainty quantification



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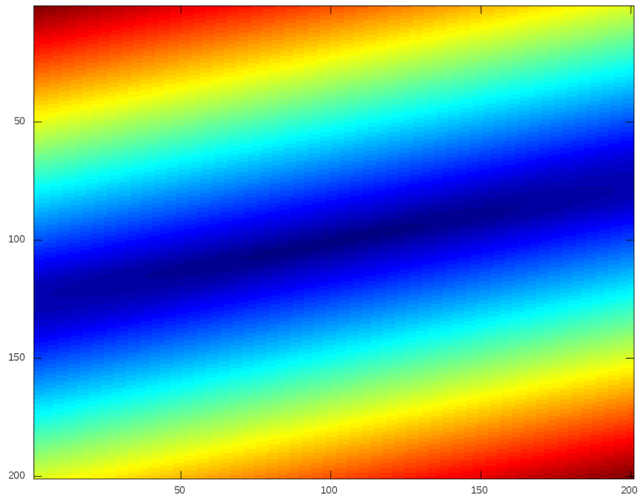
Mooney-Rivlin machine

- Inverse Methods
- Uncertainty quantification



uniaxial

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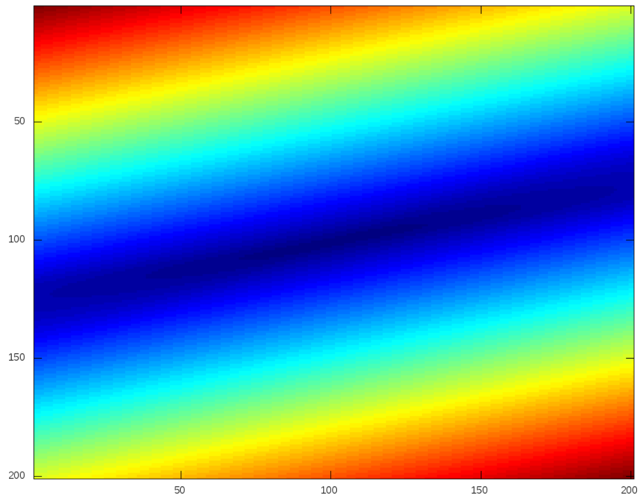
- Inverse Methods
- Uncertainty quantification

Mooney-Rivlin machine



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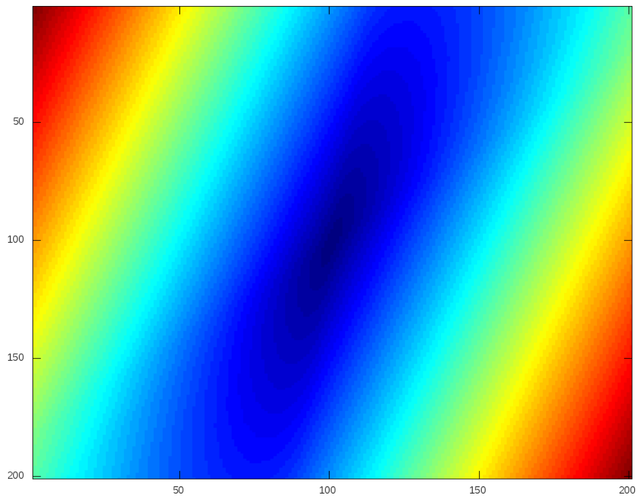

biaxial

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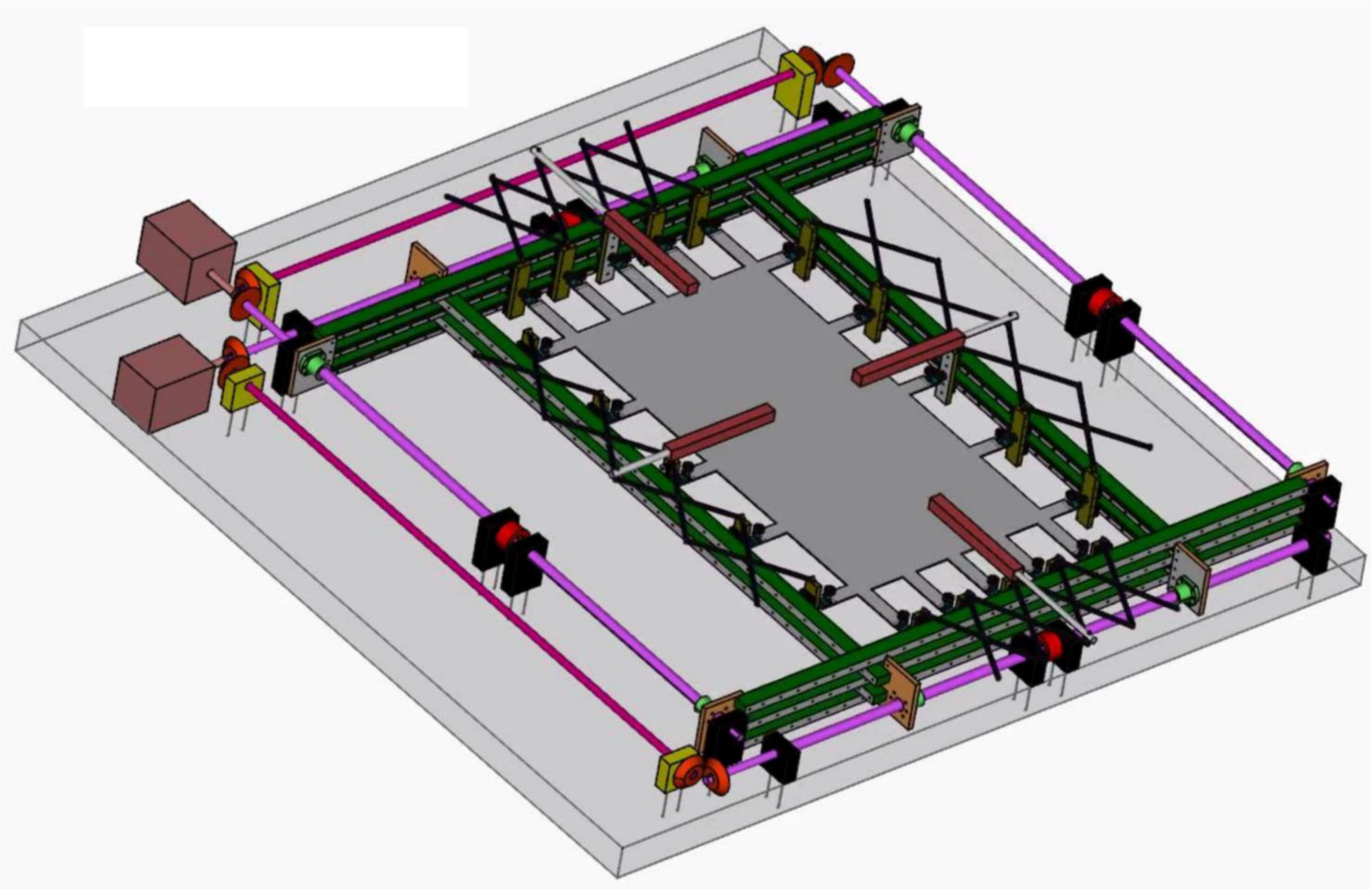
Universal elasticity modeling machine!



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- Universal elasticity
  - Theory
  - Measurement machine

Uses

Simulation

All kinds of materials

All kinds of material models

Material formulation

Application engineering

Quality / inspection

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Universal Plasticity

What phenomena should be covered?

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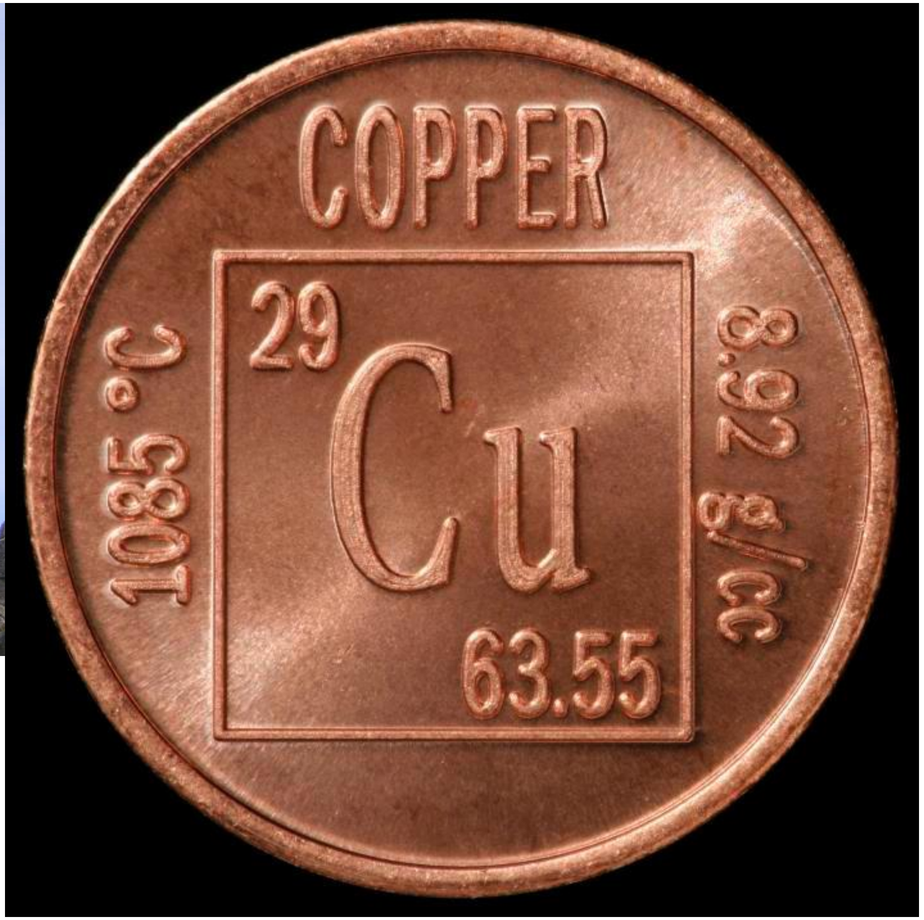

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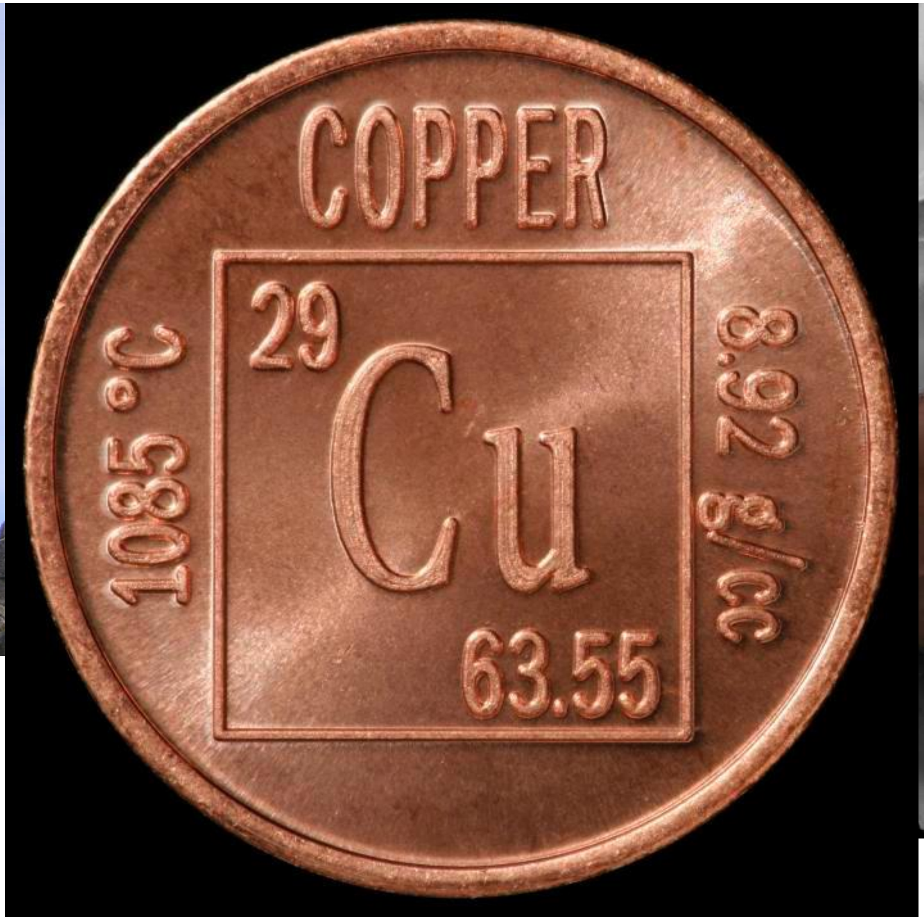

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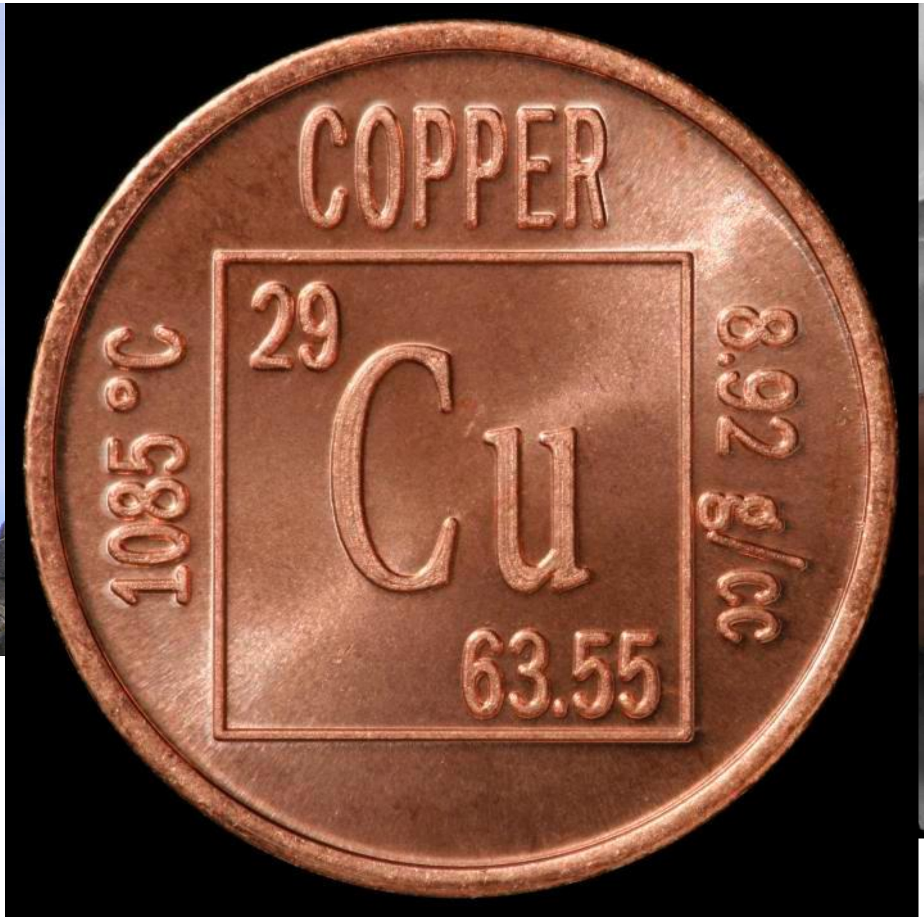

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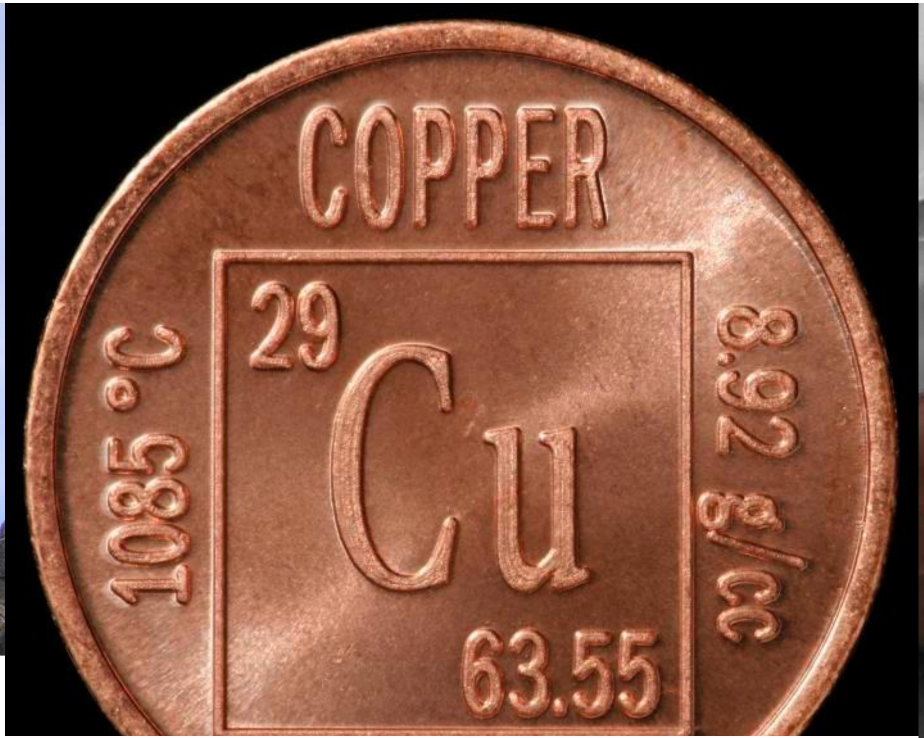

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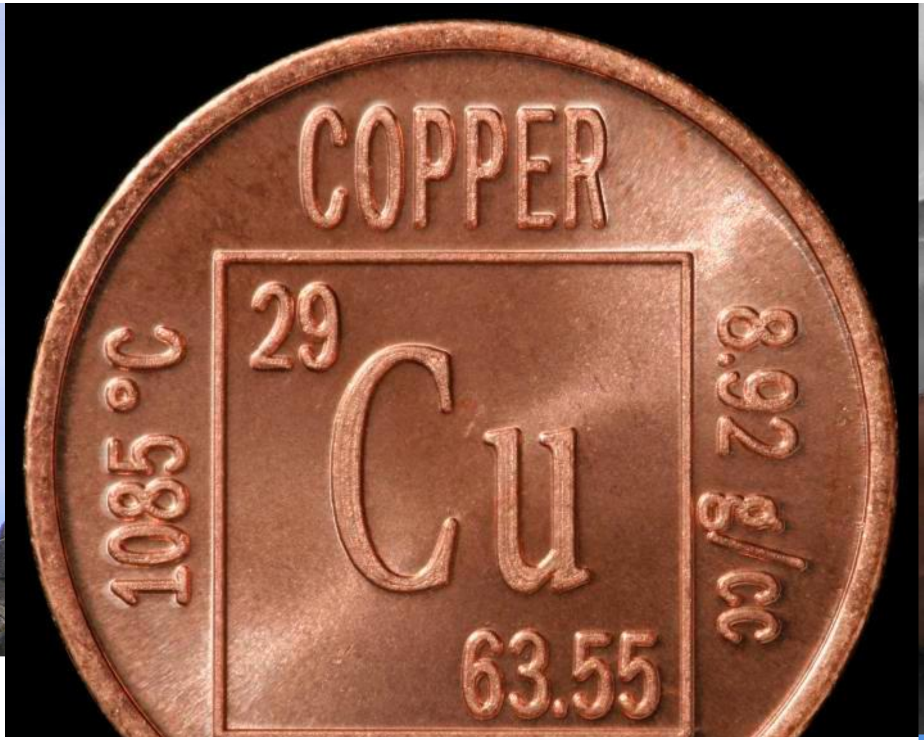
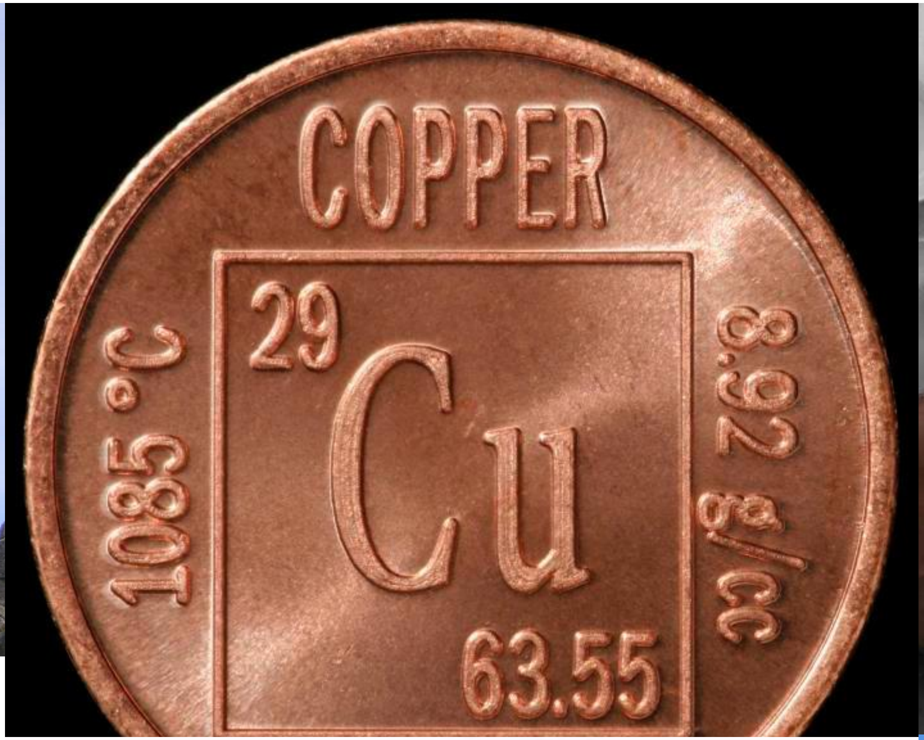


“There are no states of matter”




“There are no states of matter”

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Locally relaxed configuration

$$\rho_M \frac{\partial^2 \phi}{\partial t^2} = \nabla \cdot P(\nabla_X \phi, \kappa) + \rho_M b$$

$$P(F, \kappa) = \psi\left(F \kappa F^T\right) F\, \kappa / \sqrt{|\kappa|}$$

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where $a(F, \kappa) = P\Gamma(\Sigma)P^T$ wherein $F\kappa F^T = V\Sigma^2 V^{-1}$ and $P = F^{-1}V\Sigma$.

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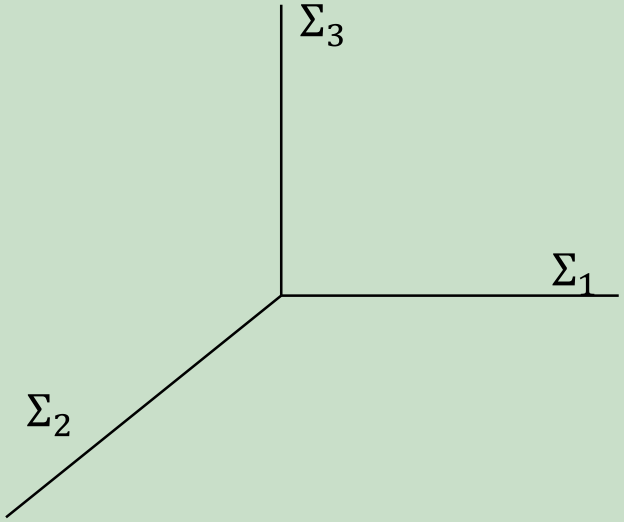

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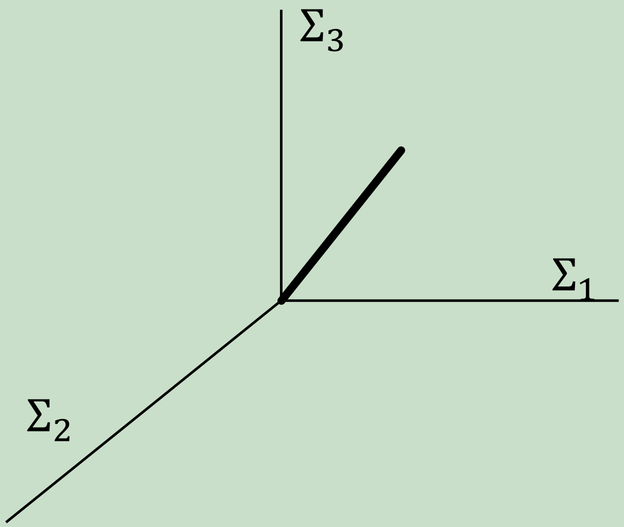

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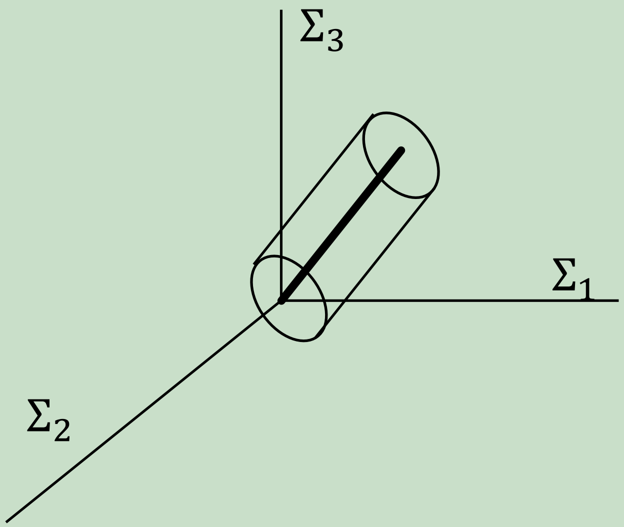

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Eulerian universal plasticity

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Space Isotropy

Locally relaxed configuration

Plasticity

$$\rho\left(\frac{\partial v}{\partial t} + v \cdot (\nabla_x v)\right) = \nabla \cdot \sigma(\omega) + \rho b$$

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho v) = 0$$

$$\frac{\partial \omega}{\partial t} + v \cdot (\nabla_x \omega) = (\nabla_x v)\omega + \omega(\nabla_x v)^T + m(\omega)$$

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Quick dissipation

Incompressibility

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$$\omega = I + \delta\omega' + \delta\omega''$$

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$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho v) = 0$$

$$0 = (\nabla_x v) + (\nabla_x v)^T + A\omega''$$

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$$A\omega'' = -[(\nabla_x v) + (\nabla_x v)^T]$$

$$\omega = I + \delta\omega' + \delta\omega''$$

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$$\rho\left(\frac{\partial v}{\partial t} + v\cdot(\nabla_x v)\right) = \nabla\cdot\left(B\omega' - \frac{C}{A}[(\nabla_x v) + (\nabla_x v)^T]\right) + \rho b$$

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$$\rho\left(\frac{\partial v}{\partial t} + v \cdot (\nabla_x v)\right) = \nabla \cdot \left(-IP - \frac{C}{A}[(\nabla_x v) + (\nabla_x v)^T]\right) + \rho b$$

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Beyond current purview?

- Anisotropy
- Hardening
- Fracture

- Chemical / material change
- Temperature

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Universal Fatigue

(?)

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Universal Fatigue

- Universal fracture (?)
- Universal hardening (?)
- Breakage?

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Universal Fatigue

- Universal fracture (?)
- Universal hardening (?)
- Breakage?

- Markov property

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THE TRUTH IS OUT THERE